$$e^x = 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!} + \frac{x^{n+1}}{(n+1)!}e^c \tag{1.11}$$

$$\begin{aligned}\sin(x) &= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots + (-1)^{n-1}\frac{x^{2n-1}}{(2n-1)!} \\ &\qquad + (-1)^n \frac{x^{2n+1}}{(2n+1)!}\cos(c)\end{aligned} \tag{1.12}$$

$$\begin{aligned}\cos(x) &= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots + (-1)^n \frac{x^{2n}}{(2n)!} \\ &\qquad + (-1)^{n+1}\frac{x^{2n+2}}{(2n+2)!}\cos(c)\end{aligned} \tag{1.13}$$

$$\frac{1}{1-x} = 1 + x + x^2 + \cdots + x^n + \frac{x^{n+1}}{1-x}, \qquad x \neq 1 \tag{1.14}$$

$$\begin{aligned}(1+x)^\alpha &= 1 + \binom{\alpha}{1}x + \binom{\alpha}{2}x^2 + \cdots \\ &\qquad + \binom{\alpha}{n}x^n + \binom{\alpha}{n+1}x^{n+1}(1+c)^{\alpha-n-1}\end{aligned} \tag{1.15}$$

In this last formula, α is any real number. The coefficients $\binom{\alpha}{k}$ are called *binomial coefficients* and are defined by

$$\binom{\alpha}{k} = \frac{a(a-1)\dots(a-k+1)}{k!}, \qquad k = 1, 2, 3, \dots$$

In all of the formulas, except (1.14), the point c is between 0 and x. The proof of (1.14) is taken up in problem 8.

By rearranging the terms in (1.14), we obtain the sum of a finite geometric series or progression,

$$1 + x + x^2 + \cdots + x^n = \frac{1 - x^{n+1}}{1-x}, \qquad x \neq 1 \tag{1.16}$$

And by letting $n \to \infty$ in (1.14) when $|x| < 1$, we obtain the infinite *geometric series*

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots = \sum_{j=0}^{\infty} x^j, \qquad |x| < 1 \tag{1.17}$$

EXAMPLE Approximate $\cos(x)$ for $|x| \leq \pi/4$, with an error of no greater than 10^{-5}. Since the point c in the remainder of (1.13) is unknown, we consider the worst possible case