Wielomian Taylora

dana jest funkcja $f(x)$ - ciągła, różniczkowalna

$n=1: \; p_1(x)$

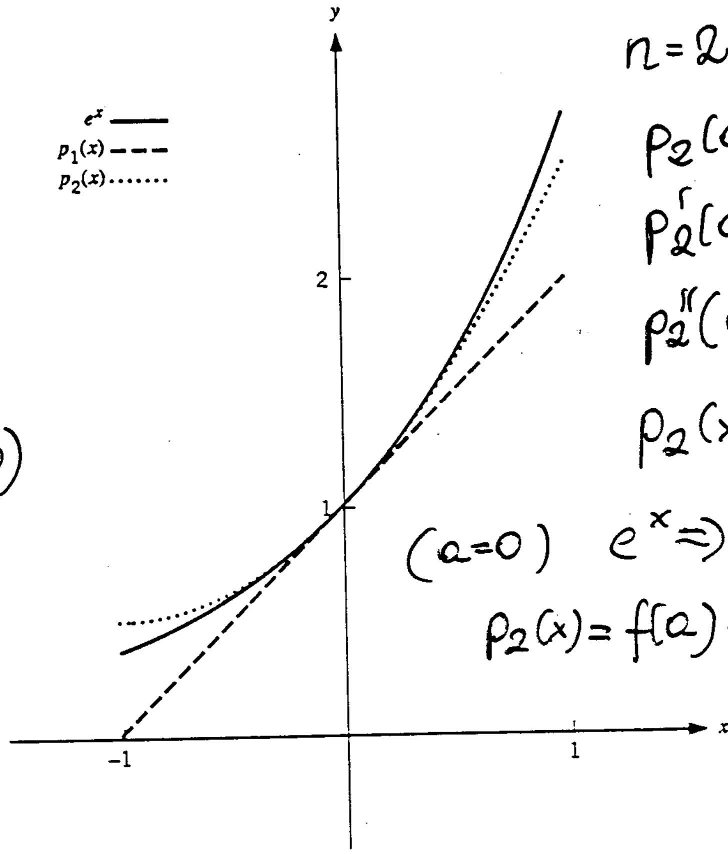
$p_1(a) = f(a)$

$p_1'(a) = f'(a)$

$p_1(x) = f(a) + (x-a)f'(a)$

$p_1(x) = b_0 + b_1 x$

$e^x \Rightarrow 1 + x \quad (a=0)$



$n=2: \; p_2(x)$

$p_2(a) = f(a)$

$p_2'(a) = f'(a)$

$p_2''(a) = f''(a)$

$p_2(x) = b_0 + b_1 x + b_2 x^2$

$(a=0) \quad e^x \Rightarrow 1 + x + \frac{1}{2}x^2$

$p_2(x) = f(a) + (x-a)f'(a) + \frac{1}{2}(x-a)^2 f''(a)$

Figure 1.2. Linear and quadratic Taylor approximations.

$p_n^{(j)}(a) = f^{(j)}(a)$

$j = 0, 1, \dots, n$

$$
\begin{aligned}
p_n(x) &= f(a) + (x-a)f'(a) \\
&\quad + \frac{(x-a)^2}{2!} f''(a) + \cdots + \frac{(x-a)^n}{n!} f^{(n)}(a) \\
&= \sum_{j=0}^{n} \frac{(x-a)^j}{j!} f^{(j)}(a)
\end{aligned}
\tag{1.6}
$$

In the formula, $f^{(0)}(a) = f(a)$, and

$$
j! = \begin{cases} 1, & j = 0 \\ j(j-1)\dots(2)(1), & j = 1, 2, 3, 4, \dots \end{cases}
$$

which is called "j factorial."